SOLUTION – DIFFERENTIATION

Q.1. Solve [Any 3] (2 Marks each) **(06)**

1) Let y = tan x + 4 sec x - $\sqrt{x}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\tan x + 4\sec x - \sqrt{x})$

= $\frac{d}{dx}(\tan x) + 4\frac{d}{dx}(\sec x) - \frac{d}{dx}(\sqrt{x})$

= $\sec^2 x$ + 4 sec x tan x - $\frac{1}{2\sqrt{x}}$

2) The supply S is given as S = $2P^2$

Marginal supply = $\frac{dS}{dP} = \frac{d}{dP}(2P^2)$

= 2 $\frac{d}{dP}(P^2)$ = 2 x 2P = 4P

$\therefore$ marginal supply when P = 5

= $\left(\frac{dS}{dP}\right)_{P-5}$ = 4(5) = 20

Here, the rate of change of supply with respect to the price is positive. This means that the supply increases if the price increases.

3) Let y = $5^{(x^2+1)^3}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left[5^{(x^2+1)^3}\right]$

= $5^{(x^2+1)^3}.\log 5.\frac{d}{dx}\left(x^2+1\right)^3$

= $5^{(x^2+1)^3}.\log 5 \text{ x } 3\left(x^2+1\right)^2.\frac{d}{dx}\left(x^2+1\right)$

= $5^{(x^2+1)^3}.\log 5 \text{ x } 3\left(x^2+1\right)^2.\left(2x+0\right)$

= $6x(x^2+1)^2.\ 5^{(x^2+1)^3}$ log 5.

4) y = $(\log x)^4$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\log x\right)^4$

= $4(\log x)^3.\ \frac{d}{dx}$ (log x)

= $4(\log x)^3$ x $\frac{1}{x} = \frac{4(\log x)^3}{x}$.

Q.2. Solve [Any 4] (3 Marks each) **(12)**

1) Let f(x) = $\frac{1}{\sec x+\tan x}=\frac{1}{\frac{1}{\cos x}+\frac{\sin x}{\cos x}}$

$= \frac{\cos x}{1+\sin x}$

$\therefore$ f'(x) = $\frac{d}{dx}\left(\frac{\cos x}{1+\sin x}\right)$

$= \frac{(1+\sin x)\frac{d}{dx}(\cos x)-\cos x\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$

$= \frac{(1+\sin x)(-\sin x)-\cos x(0+\cos x)}{(1+\sin x)^2}$

$= \frac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2}$

$= \frac{-1-\sin\ x}{(1+\sin x)^2}$

$= \frac{-(1+\sin x)}{(1+\sin x)^2}=\frac{-1}{1+\sin x}$

2) Let y = $\cos\left[e^{(x^2)}\right]$

$\therefore \frac{dy}{dx}=\frac{d}{dx}\left\{\cos\left[e^{(x^2)}\right]\right\}$

$= -\sin\left[e^{(x^2)}\right].\frac{d}{dx}\left[e^{(x^2)}\right]$

$= -\sin\left[e^{(x^2)}\right].e^{(x^2)}.\frac{d}{dx}(x^2)$

$= -\sin\left[e^{(x^2)}\right].e^{(x^2)}$ x 2x

$= -2x\,e^{(x^2)}\sin\left[e^{(x^2)}\right]$.

3) Given D = $\frac{P+5}{P-1}$

Marginal demand = $\frac{dD}{dP}=\frac{d}{dP}\left(\frac{P+5}{P-1}\right)$

$= \frac{(P-1)\frac{d}{dP}(P+5)-(P+5)\frac{d}{dP}(P-1)}{(P-1)^2}$

$= \frac{(P-1)(1+0)-(P+5)(1-0)}{(P-1)^2}$

$= \frac{P-1-P-5}{(P-1)^2}=\frac{-6}{(P-1)^2}$

When P = 2, the marginal demand

$= \left(\frac{dD}{dP}\right)_{P=2} = \frac{-6}{(2-1)^2} = -6$

Hence, the marginal demand is – 6, when price is 2.

4) Let $f(x) = \frac{3e^x - 2}{x^2 - 9}$

$\therefore f'(x) = \frac{d}{dx}\left(\frac{3e^x - 2}{x^2 - 9}\right)$

$= \frac{(x^2 - 9)\frac{d}{dx}(3e^x - 2) - (3e^x - 2)\frac{d}{dx}(x^2 - 9)}{(x^2 - 9)^2}$

$= \frac{(x^2 - 9)(3e^x - 0) - (3e^x - 2)(2x - 0)}{(x^2 - 9)^2}$

$= \frac{3e^x(x^2 - 9) - 2x(3e^x - 2)}{(x^2 - 9)^2}$

5) Let y = log [sin (e^x)]

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left\{\log\left[\sin\left(e^x\right)\right]\right\}$

$= \frac{1}{\sin(e^x)} \cdot \frac{d}{dx}\left[\sin(e^x)\right]$

$= \frac{1}{\sin(e^x)} x \cos(e^x) . \frac{d}{dx}(e^x)$

= cot (e^x) x e^x

= e^x. cot (e^x).

Q.3. Solve [Any 3] (4 Marks each) (12)

1) Let $f(x) = \frac{x\tan x}{\sec x + \tan x}$

$= \frac{x\left(\frac{\sin x}{\cos x}\right)}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}$

$\therefore f(x) = \frac{x\sin x}{1 + \sin x}$

$\therefore f'(x) = \frac{d}{dx}\left(\frac{x\sin x}{1 + \sin x}\right)$

$= \frac{(1+\sin x)\frac{d}{dx}(x\sin x) - x\sin x\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$

$$= \frac{(1+\sin x)\left[x\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(x)\right]-x\sin x(0+\cos x)}{(1+\sin x)^2}$$

$$= \frac{(1+\sin x)[x\cos x+\sin x(1)]-x\sin x\cos x}{(1+\sin x)^2}$$

$$= \frac{x\cos x+\sin x+x\sin x\cos x+\sin^2 x-x\sin x\cos x}{(1+\sin x)^2}$$

$$= \frac{x\cos x+\sin x+\sin^2 x}{(1+\sin x)^2}$$

$$= \frac{x\cos x+\sin x(1+\sin x)}{(1+\sin x)^2}$$

2) Let $f(x) = \frac{\log x}{x e^x}$

$\therefore f'(x) = \frac{d}{dx}\left(\frac{\log x}{x e^x}\right)$

$$= \frac{(x e^x).\frac{d}{dx}(\log x)-(\log x).\frac{d}{dx}(xe^x)}{(x e^x)^2}$$

$$= \frac{(x e^x).\left(\frac{1}{x}\right)-(\log x)\left[x\frac{d}{dx}(e^x)+e^x\frac{d}{dx}(x)\right]}{x^2.(e^x)^2}$$

$$= \frac{e^x-(\log x)\left(x e^x+e^x.1\right)}{x^2.(e^x)^2}$$

$$= \frac{e^x[1-(x+1)\log x]}{x^2.(e^x)^2}$$

$$= \frac{1-(x+1)\log x}{x^2 e^x}$$

3) Given $C = 1500 - 75n + 2n^2 + \frac{n^3}{5}$

Marginal Cost = MC = $\frac{dC}{dn}$

$$= \frac{dC}{dn}\left(1500-75n+2n^2+\frac{n^3}{5}\right)$$

$$= \frac{dC}{dn}(1500)-75\frac{d}{dn}(n)+2\frac{d}{dn}n^2+\frac{1}{5}\frac{d}{dn}(n^3)$$

$= 0 - 75(1) + 2(2n) + \frac{1}{5}(3n^2)$

$= -75 + 4n + \frac{3n^2}{5}$

When n =10, the marginal cost

$= \left(\frac{dC}{dn}\right)_{n=10} = -75 + 4(10) + \frac{3}{5}(10)^2$

= - 75 + 40 + 60 = 25.

Hence, the marginal cost is 25at n = 10.

4) Given S = P^2 + 9P- 2.

$\therefore$ Marginal supply $= \frac{dS}{dP}$

$= \frac{d}{dP}(P^2 + 9P - 2)$

$= \frac{d}{dP}(P^2) + 9\frac{d}{dP}(P) - \frac{d}{dP}(2)$

= 2P + 9(1) – 0

= 2P + 9

When P = 7, marginal supply

$= \left(\frac{dS}{dP}\right)_{P=7} = 2(7) + 9$

= 14 + 9 = 23.

Hence, the marginal supply is 23, when price is 7.

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